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AN EXACT TREATMENT OF A RECTANGULAR WAVEGUIDE SYMMETRICALLY LOADED WITH RESISTIVELY COATED DIELECTRIC SLABS FOR MAXIMUM ATTENUATION

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ABSTRACT

An exact dispersion relation is presented for a waveguide with resistively-coated dielectric slabs parallel to, and against, either pair of guide walls. The current in the resistive films is assumed to be a sheet current. The accuracy of such approximations is demonstrated by measurements of insertion loss through an experimental structure. Agreement is shown to be particularly good when the surface resistance is optimum for greatest attenuation.

INTRODUCTION

Resistive metallic coatings for attenuating undesirable waveguide modes propagating in the beam pipes of particle accelerators have an advantage over the more usual damping materials, such as ferrites, in that they can be employed inside the focusing magnetic fields. Whilst the transverse location of a film is constrained so as not to obstruct the beam, the film resistance is free to be optimized for highest loss in a particular band. To do this, the modes supported in a waveguide structure such as that shown in fig. 1 were studied. The approximate power loss method is not capable of providing the optimum loss, hence here the structure will be solved exactly. (N.B. An elaborate transverse resonance method [1] is another approach and is particularly useful for several films.)

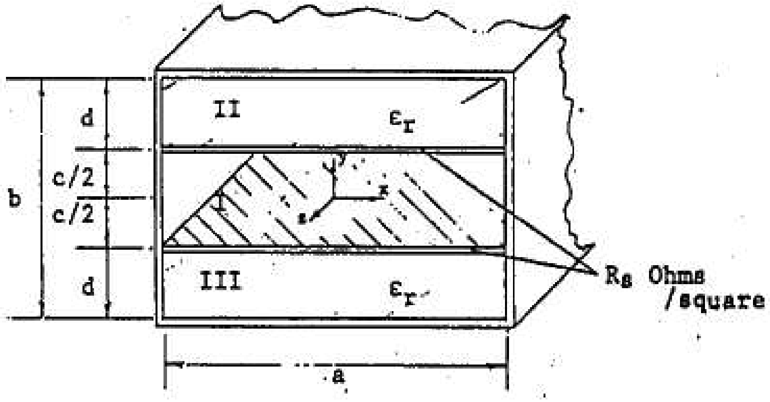

Figure 1: A dielectric-loaded waveguide with resistive coatings on the air-dielectric interfaces of each slab.

EXACT TREATMENT

The structure in fig. 1 need not necessarily have dimension b less than a. This treatment is thus applicable to both E- and H-plane[1)] loaded waveguides. A field description in terms of hydrid modes (five-component fields) [2], [4] was found to satisfy this structure. These hybrid modes are of two types: one with H_y, the y-component of $\underline{H}$, equal to zero, so-called LSM (longitudinal section magnetic) mode, with all field components therefore derivable from E_y, the y-component of $\underline{E}$; the other having $E_y = 0$, the so-called LSE (longitudinal section electric) mode, with all other field components determined from H_y.

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1) The terms E- and H-planes refer to the planes of $\underline{E}$ and $\underline{H}$ in the dominant mode of an empty guide.

If a separable solution is assumed, then by the continuity of components of $\underline{E}$ tangential to the dielectric surface, the fields will have identical x- and z-dependences in the three regions. Hence a field parameter may be written:

$$\Psi e^{j\omega t} = \{A \sin(k_x x) + B \cos(k_x x)\}\{C \sin(k_y y) + D \sin(k_y y)\}e^{j(\omega t - k_z z)}, \quad (1)$$

where $\Psi = E_y$ for LSM fields or $\Psi = H_y$ for LSE fields. It should be noted that k_y, k_z and coefficients C and D may be complex. C, D and k_y take the appropriate subscript (I, II or III, see fig. 1) according to the position of the field point because the y-dependence is clearly not identical in the three regions.

If the metallic walls at $x = \pm a/2$ are considered, i.e. making k_x take discrete values, the separation equation may be written:

$$k_{yI} = \{\omega^2 \varepsilon_0 \mu_0 - (m\pi/a)^2 - k_z^2\}^{1/2} \quad \text{and} \quad k_{yII} = \{\omega^2 \varepsilon_0 \mu_0 \varepsilon_r - (m\pi/a)^2 - k_z^2\}^{1/2}, \quad (2)$$

where m is an integer and ω is the angular frequency of the wave.

The metallic walls at $y = \pm b/2$ require $E_z = 0$. For LSE we have E_z in terms of H_y by:

$$E_z = - \frac{j\omega\mu_0}{k_x^2 + k_z^2} \cdot \frac{\delta H_y}{\delta x} , \quad (3)$$

and for LSM we have:

$$E_z = - \frac{1}{j\omega\varepsilon_0\varepsilon_r} \cdot \frac{\delta H_x}{\delta x} , \quad (4)$$

but

$$H_x = - \frac{j\omega\varepsilon_0\varepsilon_r}{k_x^2 + k_z^2} \cdot \frac{\delta E_y}{\delta z} . \quad (5)$$

It only remains for the dielectric - air interfaces with their resistive films to be accommodated. If an isotropic and very thin film is assumed, i.e. one for which the skin depth is much greater than the thickness, a square resistance, R_s, can characterize the film. The consequent sheet current flowing in response to the tangential electric field causes a discontinuity in magnetic field components tangential to the interface such that:

$$H_{x_-} - H_{x_+} = \frac{E_z}{R_s} , \quad (6)$$

and

$$H_{z_+} - H_{z_-} = \frac{E_x}{R_s} , \quad (7)$$

where the subscripts + and - denote the field points just above and below the interface, respectively. These boundary conditions (actually, only one is needed for isotropic films) can be written in terms of the original field parameters. For LSM modes, equation (7) becomes:

$$j\omega\varepsilon_0\{\varepsilon_{r-}E_{y-} - \varepsilon_{r+}E_{y+}\} = \frac{1}{R_s} \cdot \frac{\delta E_y}{\delta y} , \qquad (8)$$

and for LSE modes, equation (7) becomes;

$$\frac{-1}{j\omega\mu_0}\left\{\frac{\delta H_{y-}}{\delta_y} - \frac{\delta H_{y+}}{\delta y}\right\} = \frac{1}{R_s} H_y \qquad (9)$$

with $+\rightarrow II$ and $-\rightarrow I$ at $y = +c/2$ and with $+\rightarrow I$ and $-\rightarrow II$ at $y = -c/2$.

With regard to the continuity of the normal component of electric displacement, it should be realized that equation (8) states precisely this in the limit of infinite surface resistance.

Eliminating the coefficients, it is found that, for each mode type, the constraint on k_z for the consistency of the boundary conditions may be satisfied in two separate ways: one if $C_{II} \neq -C_{III}$ and the other if $C_{II} \neq C_{III}$. That is, the eigenvalue equations for LSM and LSE modes both factorise into two. The two solution sets correspond to non-anti-symmetric and non-symmetric fields with respect to y.

If the separation equation (2) is used to determine k_{yI} and k_{yII} from k_z , the following conditions for consistency become the dispersion equations for the various modes.

For LSM, non-antisymmetric ($C_{II} \neq -C_{III}$):

$$k_{yI}\varepsilon_r \cot(k_{yII}d) = -k_{yII}\left\{\cot(k_{yI}\frac{c}{2}) + \frac{jk_{yI}}{R_s\omega\varepsilon_0}\right\} \qquad (10)$$

For LSM, non-symmetric ($C_{II} \neq C_{III}$):

$$k_{yI}\varepsilon_r \cot(k_{yII}d) = k_{yII}\left\{\tan(k_{yI}\frac{c}{2}) - \frac{jk_{yI}}{R_s\omega\varepsilon_0}\right\} \qquad (11)$$

For LSE, non-antisymmetric ($C_{II} \neq -C_{III}$):

$$k_{yII}\cot(k_{yII}d) = k_{yI}\tan(k_{yI}\frac{c}{2}) - \frac{j\omega\mu_0}{R_s} \qquad (12)$$

For LSE, non-symmetric ($C_{II} \neq C_{III}$):

$$k_{yII}\cot(k_{yII}d) = k_{yI}\cot(k_{yI}\frac{c}{2}) - \frac{j\omega\mu_0}{R_s} \qquad (13)$$

For example, the TE^{10} like mode, for $b > a$, in an E-plane loaded guide is actually a solution to equation (12). If, for $R_s \rightarrow \infty$ equation (10) is combined with (11) and (12) with (13) then they become identical to thedispersion equations presented in [2] for symmetric dielectrics adjacent to the wall.

NUMERICAL RESULTS

Figs. 2 and 3 show the imaginary part of k_z satisfying the LSE equation (12) for a particular guide (with cut-off at 1 GHz when empty) with E-plane slabs and films. These are believed to be the least attenuated modes supported. The optimum resistances are evident. The attenuation achievable is strongly enhanced by having films displaced away from the wall. If the film resistance is

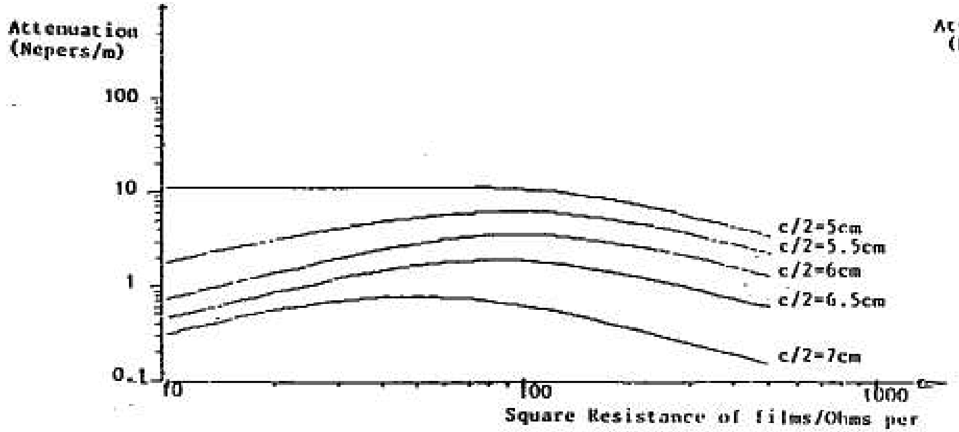

Figure 2: Attenuation due to E-plane resistive films at various positions in a 15x7.5cm waveguide at 1.4GHz.

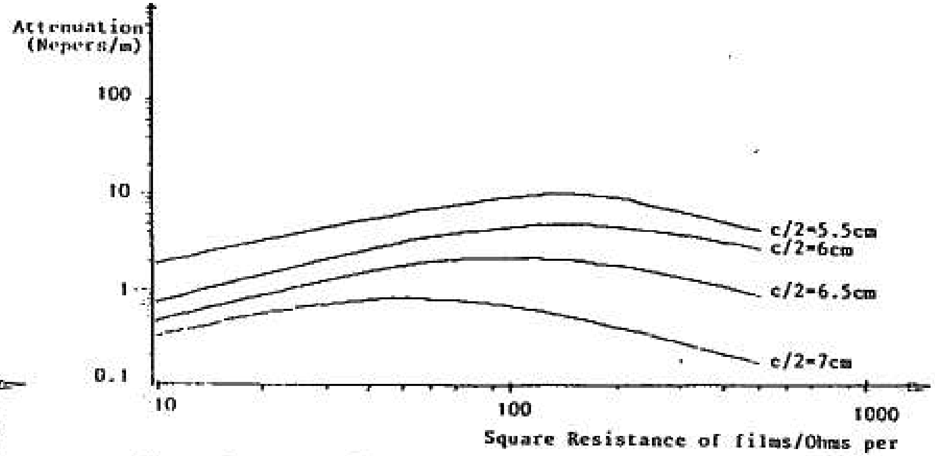

Figure 3: Attenuation due to E-plane resistive film coatings on Alumina (ϵ_r=9.6) slabs of various thickness at 1.4GHz.

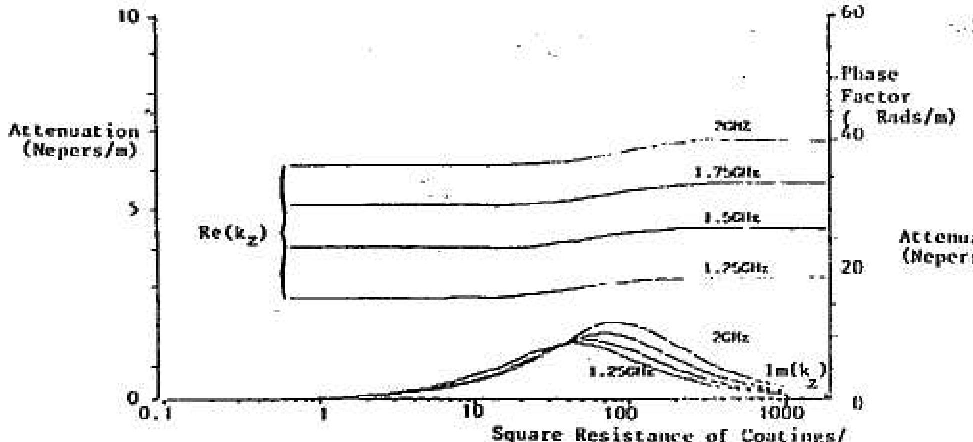

Figure 4: Propagation Coefficients for a 15x7.5cm waveguide loaded with Alumina(ϵ_r=9.6) H-plane slabs, thickness 5mm, with resistive coatings.

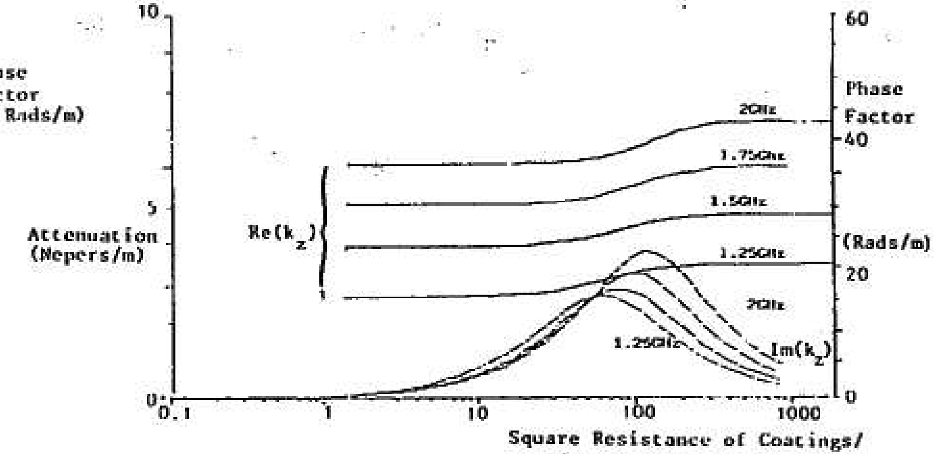

Figure 5: Propagation Coefficients for a 15x7.5cm waveguide, loaded with H-plane dielectric slabs (ϵ_r=3.75), thickness 1cm, coated with resistive films.

high (i.e. greater than optimum), then the attenuation is further improved by loading the side sections with dielectric. This suggests that the majority of power flows along the central section I, provided it is not below cut-off as in the upper curve of fig. 2. The attenuations for high resistance cases were found to be asymptotic to Power Loss Method estimates [3].

Figs. 4 and 5 show the solutions to some H-plane loaded structures. The attenuated modes are given by the LSM equation (10). Their comparison reveals the dependence on film positions to be stronger than the influence of dielectric permittivity, although it is the dielectric which causes the E field components responsible for the losses. The phase factors undergo a transition at the optimum but otherwise they are nearly constant, suggesting that, for resistances an order of magnitude away from the optimum, a power loss method is reasonably accurate. The frequency dependence of the loss reverses near the optimum case.

EXPERIMENTAL VERIFICATION

Figs. 6, 7, 8 and 9 show, as the solid lines, the attenuation observed between two launchers in an L-band waveguide loaded, for part of its length, with various dielectric slabs and conductive films (10 µm thick carbon films on thin alumina substrates). The launching antennas were well matched to the guide and the return loss was measured to be weaker than 20 dB, hence the observed transmission loss should correspond to the attenuation predicted by equation (12), for E-plane loading, or equation (10), for H-plane loading (predictions are plotted as circled dots). This assumes that no energy is carried by attenuated higher modes. This is the case over a limited band. The cut-off frequency of the loaded section is obscured by the higher cut-off of the preceding empty guide sections. In a lossy waveguide the cut-off is not well defined but rather a smooth transition.

These results and those for several other structures gave good agreement; particularly near optimum resistance and also for E-plane films with no loading, provided there exists a good electrical contact between the edge of the film and the guide wall. Low-resistance films (<20 Ω/square) in the H-plane configuration do, however, support more than one mode of low attenuation; one with power flowing mainly through the dielectric and another via the guide centre, giving a complicated interference. Where the fit is poor, the equations under-estimate the loss. The effect of higher order modes is, in general, to increase losses.

CONCLUSION

The dispersion equations (10), (11), (12), (13) together with (2) are analytically consistent for extremes of R_s with the more familiar modes of the appropriate lossless structures. Numerically, their solutions converge to Power Loss Method estimates of attenuation in the limit of large R_s. Experimentally, the LSE solutions are well supported for all the E-plane film resistances tested. The LSM solutions closely predict the attenuation due to an H-plane film for resistances near and above optimum. The experiment further suggests that more attenuation is available from a film which is insulated from the guide walls by an air gap. Such films may be readily deposited on parts of the substrate of a micro-strip circuitto prevent cavity modes, thus allowing it to be enclosed.

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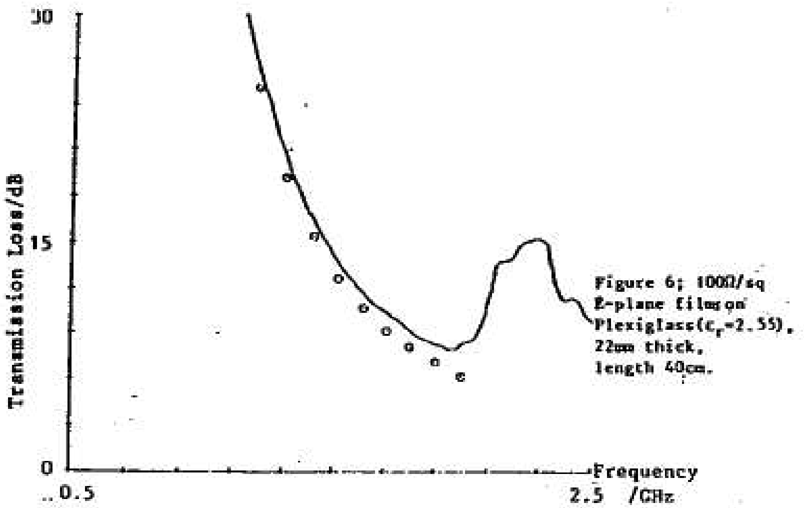

Figure 6; 100Ω/sq E-plane film on Plexiglass(ϵ_r=2.55), 22mm thick, length 40cm.

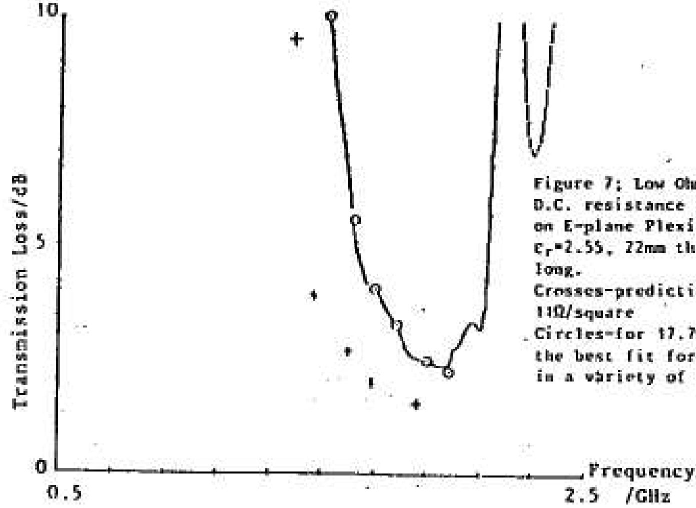

Figure 7; Low Ohmic film, D.C. resistance 11Ω/square, on E-plane Plexiglass slab, ϵ_r=2.55, 22mm thick, 36cm long.
Crosses-predictions for 11Ω/square
Circles-for 17.7Ω/square, the best fit for this film in a variety of structures.

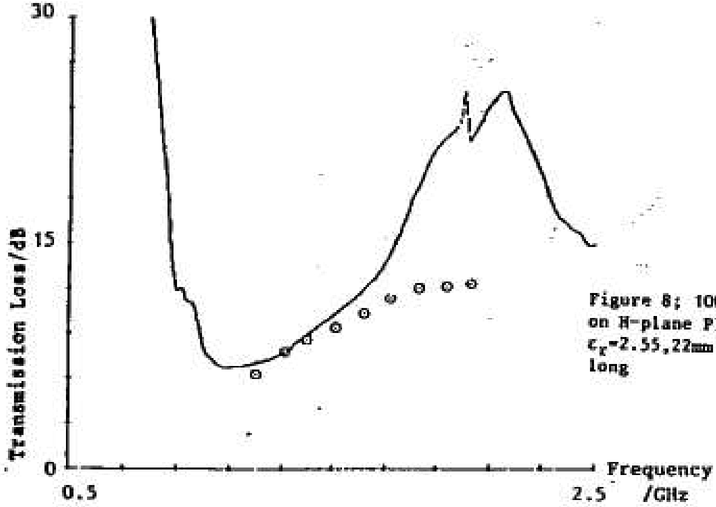

Figure 8; 100Ω/square films on H-plane Plexiglass slabs, ϵ_r=2.55,22mm thick, 18cm long

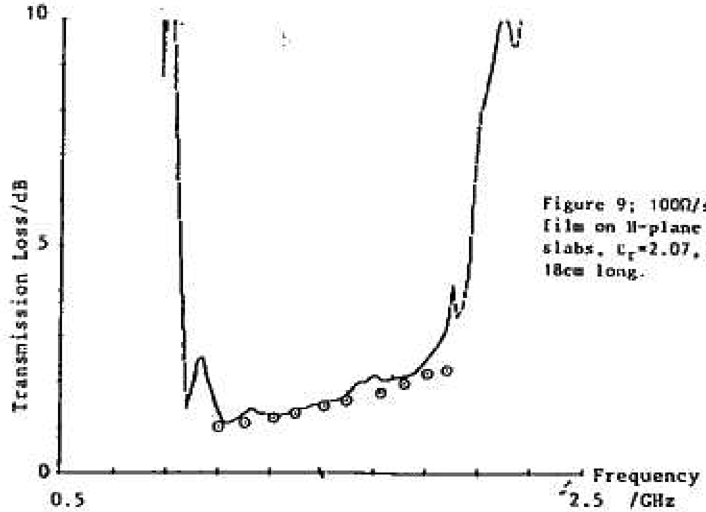

Figure 9; 100Ω/square film on H-plane Teflon slabs, ϵ_r=2.07, 9mm thick, 18cm long.

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