Calculation of electron current from rest gas ionization in LHC, SPS, PS and LEIR

2013.10.09 mariusz.sapinski@cern.ch
2013.12.04 general revision, proper units, adding LEIR for Alexandr, etc.
2014.01.28 restructurized
2014.05.13 corrected values for PS (ions Pb54+)
2015.03.20 some refactoring

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1. Initialization cells

1.1 Physics constants

In[1]:=
```
ℏ = Quantity[1.054*^-34, "Joules" * "Seconds"];
m = UnitConvert[Quantity["ElectronMass"]];
c = UnitConvert[Quantity["SpeedOfLight"]];
N_A = UnitConvert[Quantity["AvogadroConstant"]];
R = UnitConvert[Quantity["MolarGasConstant"]];
```

1.2 Physical parameters

1.2.1 Beam particles

In[6]:=
```
zprot = 1;
(* mprot = Quantity[0.938272*10^9,"Electronvolts"]*)
mprot = UnitConvert[Quantity["ProtonMass"] * c^2, "Electronvolts"];
zPb = 82;
zPb54 = 54;  (*Pb 54+ ionization state for LEIR and for PS *)
mPb = UnitConvert[Quantity[207.97665, "AtomicMassUnits"] * c^2, "Electronvolts"];
(* isotope 208Pb *)
zAr11 = 11; (* Ar 11+ ionization state for LEIR *)
mAr = UnitConvert[Quantity[39.948, "AtomicMassUnit"] * c^2, "Electronvolts"];
(* isotope 40Ar, 99% abundance *)
```

1.2.2 Rest gas

```
(* gas pressure in the chamber *)
p = UnitConvert[Quantity[10*^-8, "Millibars"]];
(* gas temperature, shall it be higher? *)
T = Quantity[300, "Kelvins"];
```

1.2.3 Beam parameters

```
(* particles per bunch *)
Symbolize[Npart];
Npart = {12*^11, 1.3*^11, 1.5*^11,
   1.5*^11, 3*^8, 2.5*^8, 1.8*^10, 1.8*^10, 1.*^9, 1.*^9};
(* bunch length *)
lbunchLHC = Quantity[1.1, "NanoSeconds"];
lbunchPSinj = Quantity[100, "NanoSeconds"];
lbunchLEIR = Quantity[200, "NanoSeconds"];
Lbunch = {lbunchPSinj, lbunchLHC, lbunchLHC, lbunchLHC,
  lbunchPSinj, lbunchLHC, lbunchLHC, lbunchLHC, lbunchLEIR, lbunchLEIR}
```

Out[20]= {100 ns, 1.1 ns, 1.1 ns, 1.1 ns, 100 ns, 1.1 ns, 1.1 ns, 1.1 ns, 200 ns, 200 ns}

Comment: LEIR bunch length at injection is probably longer.

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2. Investigated cases

Cases are :

1. PS, protons at injection 1.4 GeV; PS.p.Inj
2. PS, protons at extraction 26 GeV, PS.p.Ext
3. SPS, protons at extraction 450 GeV, SPS.p.Ext
4. LHC, protons at flat top 7 TeV, LHC.p.Ext
5. PS, Pb ions at injection 0.55 GeV/nucleon, PS.Pb82 + .Inj (shall be 72.2 MeV/nucleon!) (shall be Pb54+)
6. PS, Pb ions at extraction 10.25 GeV/nucleon, PS.Pb82 + .Ext (shall be 5.9 GeV/nucleon!) (shall be Pb54+)
7. SPS, Pb ions at extraction, 177.4 GeV/nucleon, SPS.Pb82 + .Ext
8. LHC, Pb ions at flat top, 2.76 TeV/nucleon, LHC.Pb82 + .Ext
9. LEIR, Argon 11 + ion, LEIR.Ar11, at injection Ek = 4.2 MeV/n, using LEIR.Ar11+.Inj
10. LEIR, Lead 54 + ions, extraction energy: 72 MeV/nucleon, using LEIR.Pb54 +.Ext

In[21]:=

In[22]:=
```
caseName =
  {"PS.p.Inj", "PS.p.Ext", "SPS.p.Ext", "LHC.p.Ext", "PS.Pb54+.Inj", "PS.Pb54+.Ext",
   "SPS.Pb82+.Ext", "LHC.Pb82+.Ext", "LEIR.Ar11+Inj", "LEIR.Pb54+Inj"};
```

In[23]:=
```
(* list of ionization states *)
Z = {zprot, zprot, zprot, zprot, zPb54, zPb54, zPb, zPb, zAr11, zPb54};
```

In[24]:=
```
(* list of atomic masses in eV *)
mass = {mprot, mprot, mprot, mprot, mPb, mPb, mPb, mPb, mAr, mPb};
```

In[25]:=
```
(* list of kinetic energies of projectiles (beam) *)
Ek = {Quantity[1.4, "GigaElectronvolts"],
  Quantity[26, "GigaElectronvolts"], Quantity[450, "GigaElectronvolts"],
  Quantity[7, "TeraElectronvolts"], 208 * Quantity[72.2, "MegaElectronvolts"],
  208 * Quantity[5.9, "GigaElectronvolts"],
  208 * Quantity[177.4, "GigaElectronvolts"],
  208 * Quantity[2.76, "TeraElectronvolts"],
  40 * Quantity[4.2, "MegaElectronvolts"], 208 * Quantity[72, "MegaElectronvolts"]}
```

Out[25]=
```
{1.4 GeV, 26 GeV, 450 GeV, 7 TeV, 15 017.6 MeV,
 1227.2 GeV, 36 899.2 GeV, 574.08 TeV, 168. MeV, 14 976 MeV}
```

3. Ionizationcross-section

3.1 Theoreticalexplanation

Ionization cross-section can be estimated using formula from[1]

```
σ = 4 π Z^2 (ħ / mc)^2 β^-2 (M^2 x + C);
```

where :

Z is charge of the impacting particle

(1 for protons, 82 for fully ionized Pb ions, for LEIR - partly charges state)

ħ is a reduced planck constant

m is electron mass

β is impacting particle velocity

M and C are coefficients determined by experiment, dependig of gas type

$x = \ln\left(\beta^2 / \left(1 - \beta^2\right)\right) - \beta^2$

The above formula was checked for protons at energies 20 MeV - 385 GeV. Because of relaying on Bethe theory it shall be true for fully ionized Pb probably even at PS injection energy 1.4 GeV / nucleon. What is probably not a very good approximation is LEIR case with partly - ionized Argon and Lead with energy as low as 4.2 MeV / nucleon. No apropriate literature was found. Maybe it could be measured?

3.2 calculaterelativisticβ :

Syntax::sntxi: Incomplete expression; more input is needed .

In[26]:=
```
β[Ek_, mass_] := (1 - (Ek / mass + 1) ^ -2) ^ (1. / 2)
beta = β[Ek, mass]
```

Out[27]=
```
{0.915961, 0.999393, 0.999998, 1.,
 0.372435, 0.990662, 0.999986, 1., 0.0947034, 0.371974}
```

In[28]:=
```
{0.9159610123308375`, 0.9993932359134868`,
 0.9999978353193408`, 0.9999999910191992`, 0.3724351132987941`,
 0.9906621473957741`, 0.9999863610779718`, 0.999999943098796`,
 0.09470335911273271`, 0.3719738248268274`}
```

Out[28]=
```
{0.915961, 0.999393, 0.999998, 1.,
 0.372435, 0.990662, 0.999986, 1., 0.0947034, 0.371974}
```

3.2b relativistic gamma γ_{rel}:

In[29]:=
```
γrel[b_] := 1. / (1 - b^2) ^0.5
gammarel = γrel[beta]
```

Out[30]=
```
{2.4921, 28.7105, 480.605, 7461.52,
 1.07752, 7.33462, 191.468, 2964.31, 1.00451, 1.0773}
```

3.2c checking: gamma*beta (for normalized emittance):

In[31]:=
```
beta * gammarel
```

Out[31]=
```
{2.28267, 28.6931, 480.604, 7461.52,
 0.401306, 7.26613, 191.466, 2964.31, 0.0951309, 0.400729}
```

3.3 calculate x :

x is a auxiliary variable for cross section formula

In[32]:=
```
x[β_] := Log[β^2 / (1 - β^2)] - β^2
ixy = x[beta]
```

Out[33]=
```
{0.811708, 5.71453, 11.3501, 16.835,
 -1.96477, 2.98504, 9.50944, 14.9888, -4.71397, -1.96731}
```

3.4 calculate cross section:

Values from[2] which describes ionization by high-energy electrons but claims to be universal (based on Bethe theory).

In[34]:=
```
(* M and C1 should be dimensionless *)
(* M=4.22^0.5; *)    (* original from Macin: this is for Argon *)
(* C1=37.93; *)      (* original from Macin: this is for Argon *)

(* for neon *)

M = 2.02^0.5;
C1 = 18.2;

(* for H2+D2 *)
(*
M=0.695^0.5;
C1=8.115;
*)
```

In[36]:=
```
cs[x_, β_, Z_] := 4 π Z^2 (ℏ / (m c)) ^2 β^-2 (M^2 x + C1)
(* σ[x_,β_, Z_]:=
 4π Z^2(PlanckConstantReduced/(ElectronMass SpeedOfLight))^2β^-2 (M^2x+C1) *)
cross = cs[ixy, beta, Z] (* output in m2 *)
```

Out[37]= $\{4.42641\times10^{-23}\ s^4J^2/(kg^2m^2),\ 5.57428\times10^{-23}\ s^4J^2/(kg^2m^2),\ 7.69844\times10^{-23}\ s^4J^2/(kg^2m^2),\ 9.77234\times10^{-23}\ s^4J^2/(kg^2m^2),\ 5.60013\times10^{-19}\ s^4J^2/(kg^2m^2),\ 1.34759\times10^{-19}\ s^4J^2/(kg^2m^2),\ 4.70856\times10^{-19}\ s^4J^2/(kg^2m^2),\ 6.10153\times10^{-19}\ s^4J^2/(kg^2m^2),\ 2.19147\times10^{-19}\ s^4J^2/(kg^2m^2),\ 5.61201\times10^{-19}\ s^4J^2/(kg^2m^2)\}$

In[38]:=
```
UnitConvert[cross, "Meters"^2]
```

Out[38]= $\{4.42641\times10^{-23}\ m^2,\ 5.57428\times10^{-23}\ m^2,\ 7.69844\times10^{-23}\ m^2,\ 9.77234\times10^{-23}\ m^2,\ 5.60013\times10^{-19}\ m^2,\ 1.34759\times10^{-19}\ m^2,\ 4.70856\times10^{-19}\ m^2,\ 6.10153\times10^{-19}\ m^2,\ 2.19147\times10^{-19}\ m^2,\ 5.61201\times10^{-19}\ m^2\}$

In[39]:=
```
UnitConvert[cross, "Barns"]
```

Out[39]= $\{442\,641.\ b,\ 557\,428.\ b,\ 769\,844.\ b,\ 977\,234.\ b,\ 5.60013\times10^9\ b,\ 1.34759\times10^9\ b,\ 4.70856\times10^9\ b,\ 6.10153\times10^9\ b,\ 2.19147\times10^9\ b,\ 5.61201\times10^9\ b\}$

4. Amount of electrons emitted when a bunch crosses zone above detector

4.1 Number density of gas

Calculate number density of target particles per unit volume
from pressure and temperature, assuming ideal gas.

In[40]:=
```
Symbolize[n]; (* gas number density in 1/meter^3*)
n = Quantity[p * N_A / (R * T)]
```

Out[41]= 2.41432×10^{15} per meter^3

4.2 Introduce depth of the active gas

In[42]:=
```
dLHC = Quantity[5, "CentyMeters"];  (* depth of the target, 5 cm *)
dLEIR = Quantity[3, "CentyMeters"]; (* depth of the target, 3 cm *)
dPS = Quantity[14.1, "MiliMeters"]; (* timepix is 14.1 mm *)
d = {dPS, dPS, dLHC, dLHC, dPS, dPS, dLHC, dLHC, dLEIR, dLEIR}
```

Out[45]=
```
{14.1 mm, 14.1 mm, 5 cm, 5 cm, 14.1 mm, 14.1 mm, 5 cm, 5 cm, 3 cm, 3 cm}
```

4.3 Number of electrons per single bunch passing through the active gas (i.e. gas above detector)

In[46]:=

In[47]:=
```
ne[n_, d_, σ_, Npart] := n * d * σ * Npart
nelec = ne[n, d, cross, Npart]
```

Out[48]=
$\{1808.2, 246.687, 1393.99, 1769.52, 5719.19, 1146.86, 1.02312\times10^{6}, 1.3258\times10^{6}, 15872.7, 40647.6\}$

4.4 Peak current

In[49]:=
```
ipeak[n_, dt_] := n * Quantity[1.6 * 10^-19, "Coulombs"] / dt
Ipeak = ipeak[nelec, Lbunch]
```

Out[50]=
$\{2.89313\times10^{-18}\ \text{C/ns}, 3.58818\times10^{-17}\ \text{C/ns}, 2.02762\times10^{-16}\ \text{C/ns}, 2.57385\times10^{-16}\ \text{C/ns}, 9.1507\times10^{-18}\ \text{C/ns}, 1.66817\times10^{-16}\ \text{C/ns}, 1.48817\times10^{-13}\ \text{C/ns}, 1.92843\times10^{-13}\ \text{C/ns}, 1.26982\times10^{-17}\ \text{C/ns}, 3.25181\times10^{-17}\ \text{C/ns}\}$

In[51]:=
```
UnitConvert[Ipeak, "Amperes"]
```

Out[51]=
$\{2.89313\times10^{-9}\ \text{A}, 3.58818\times10^{-8}\ \text{A}, 2.02762\times10^{-7}\ \text{A}, 2.57385\times10^{-7}\ \text{A}, 9.1507\times10^{-9}\ \text{A}, 1.66817\times10^{-7}\ \text{A}, 0.000148817\ \text{A}, 0.000192843\ \text{A}, 1.26982\times10^{-8}\ \text{A}, 3.25181\times10^{-8}\ \text{A}\}$

5. Summary

Final results in form of the table:

In[62]:=
```
data = {{machine, PS, PS, SPS, LHC, PS, PS, SPS, LHC, LEIR, LEIR}, {"particle", "p",
    "p", "p", "p", "Pb54+", "Pb54+", "Pb82+", "Pb82+", "Ar11+", "Pb54+"},
   Join[{energy}, Ek], Join[{ppb}, Npart], Join[{σ}, UnitConvert[cross, "barns"]],
   Join[{Nelec}, nelec], Join[{peak current}, UnitConvert[Ipeak, "NanoAmperes"]]};
Grid[data, Frame → All, Background → LighGray, ItemSize → {Automatic, 1.5},
 BaseStyle → {FontFamily → "Helvetica", 10}, FrameStyle → Thin]
```

Out[63]=

machine	PS	PS	SPS	LHC	PS	PS	SPS	LHC	LEIR	LEIR
particle	p	p	p	p	Pb54+	Pb54+	Pb82+	Pb82+	Ar11+	Pb54+
energy	1.4 GeV	26 GeV	450 GeV	7 TeV	15 017.6 MeV	1227.2 GeV	36 899.2 GeV	574.08 TeV	168. MeV	14 976 MeV
ppb	1 200 000⋱ 000 00⋱ 0	1.3×10^{11}	1.5×10^{11}	1.5×10^{11}	300 000 0⋱ 00	2.5×10^{8}	1.8×10^{10}	1.8×10^{10}	$1.\times10^{9}$	$1.\times10^{9}$
σ	442 641. b	557 428. b	769 844. b	977 234. b	5.60013×10^{9} b	1.34759×10^{9} b	4.70856×10^{9} b	6.10153×10^{9} b	2.19147×10^{9} b	5.61201×10^{9} b
Nelec	1808.2	246.687	1393.99	1769.52	5719.19	1146.86	1.02312×10^{6}	1.3258×10^{6}	15 872.7	40 647.6
current peak	2.89313 nA	35.8818 nA	202.762 nA	257.385 nA	9.1507 nA	166.817 nA	148 817. nA	192 843. nA	12.6982 nA	32.5181 nA

1 H. Ishimaru, S. Shibata, M. Inokuti, Phys. Rev. A51, 4631 (1995)

2 Rieke, Foster F., and William Prepejchal. “Ionization Cross Sections of Gaseous Atoms and Molecules for High-Energy Electrons and Positrons.” Physical Review A 6, no. 4 (1972): 1507.